(Strengthened) One-Sided Lipschitz Set-Valued Maps: I. Examples and Applications

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The one-sided Lipschitz (OSL) condition is a notion for single-valued functions and set-valued maps to weaken the Lipschitz condition and even includes certain discontinuous maps. Applications of the OSL condition can be found in approximation results of reachable sets of nonlinear control problems resp. differential inclusions. In contrast to the Lipschitz modulus the OSL constant in the (one-sided) estimate can be negative allowing a boundedness or damping effects of errors even for infinite time horizon.

We collect several examples of single-valued functions and set-valued maps for which the OSL condition hold. Among them are Lipschitz, monotone decreasing or dissipative set-valued monotone maps, linear functions (with possible negative OSL constants) and the well-known negative sign function. OSL functions can be smooth, Lipschitz continuous, they may have infinite slopes of the derivative or may be discontinuous. Many activation functions in machine learning as sigmoidal or the saturation function, the Heaviside function or the ReLU one (rectified linear unit) are OSL.

The (pointwise) negative of maximal monotone maps and the Filippov regularization of discontinuous OSL right-hand sides are further interesting examples. In applications outer and inner perturbations of OSL maps play an important role. In contrast to monotone or dissipative systems, OSL right-hand sides of differential inclusions allow non-uniqueness of solutions. The talk discusses several variants of (strengthened) OSL conditions, explain their advantages and limitations with the help of model problems in mechanical systems with Coulomb and dry friction, in electric circuits or with linear feedback together with the convergence order of Euler's method.

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